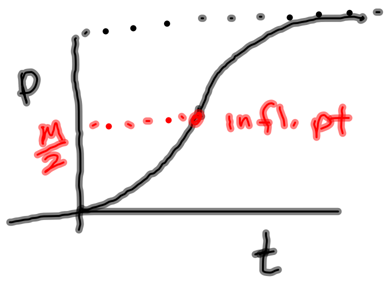
6.5 logistic growth



M = carrying capacity

$$\frac{dP}{dt} = kP(M-P)$$ d.e. for logistic growth

1. for what values of P is the ~~growth rate of P~~ close to zero?

when P is close to M

also when P is close to 0

2. For what values of P is the pop growing fastest?

when $P = \frac{M}{2}$

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How to solve $\frac{dP}{dt} = kP(M-P)$?

find P

what if $\frac{dP}{dt} = .008\,P(100-P)$

sep. var. $\frac{dP}{P(100-P)} = .008\,dt$

integrate $\int \frac{1}{P(100-P)}\,dP = \int .008\,dt$

A, B are constants

partial fractions $\int \frac{A}{P} + \frac{B}{100-P}\,dP = \int .008\,dt$ (solve for A, B)

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$$\frac{(100-P)}{(100-P)}\frac{A}{P} + \frac{B}{(100-P)}\frac{P}{P} = \frac{1}{P(100-P)} \qquad \text{find } A, B$$

common denominator

$$\frac{100A - P\cdot A + P\cdot B}{P(100-P)} = \frac{1 + 0\cdot P}{P(100-P)}$$

$A = B$

$100A = 1$

$A = \frac{1}{100} \qquad B = \frac{1}{100}$

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short cut to find A & B

$$\frac{A}{P} + \frac{B}{100-P} = \frac{1}{P(100-P)}$$

(cover up method)

to find A, set den. to zero

$P = 0$

subst

to solve for B, set it's den to zero find P, subst in right side, cover up

$P = 100$

$A = \frac{1}{100}$

$B = \frac{1}{100}$

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$$\int \frac{1/100}{P} + \frac{1/100}{100-P}\,dP = \int .008\,dt$$

$$\frac{1}{100}\int \frac{1}{P} + \frac{1}{100-P}\,dP = \int .008\,dt$$

$$\frac{1}{100}\left(\ln P - \ln(100-P)\right) = .008\,t + C$$

solve for P

$$e^{\ln\left(\frac{P}{100-P}\right)} = e^{(.8t + C)}$$

$$\frac{P}{100-P} = e^{.8t}\cdot e^{C}$$

$$\frac{P}{\cancel{100-P}} = Ce^{.8t}(100-P)$$

$$P = 100Ce^{.8t} - PCe^{.8t}$$

$$P + P\cdot Ce^{.8t} = 100Ce^{.8t}$$

$$P(\cancel{1+Ce^{.8t}}) = \frac{100Ce^{.8t}}{1+Ce^{.8t}}$$

$$P = \frac{100Ce^{.8t}}{1+Ce^{.8t}} \quad \frac{\frac{1}{Ce^{.8t}}}{\frac{1}{Ce^{.8t}}}$$

$$P = \frac{100}{1+\frac{1}{C}e^{-.8t}} \qquad A = \frac{1}{C} \qquad P = \frac{100}{1+Ae^{-.8t}}$$

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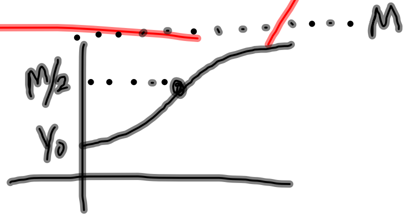
p 367 de: $\frac{dP}{dt} = kP(M-P)$

solution eqn. $P = \frac{M}{1+Ae^{-Mkt}}$

P_0 is initial value of P $\qquad A = \frac{M-P_0}{P_0}$

$\lim_{t\to\infty} P(t) = M$

graph of soln



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Ex 5

$61 = P_0$

$$\frac{dP}{dt} = .0003\,P\,(1000 - P)$$

$M = 1000$

$$A = \frac{1000 - 61}{61} = 15.394$$

soln: $$P = \frac{1000}{1 + 15.394\,e^{-.3t}}$$

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